Liquid Crystals
Publication details, including instructions for authors and subscription information:
http://www.informaworld.com/smpp/title~content=t713926090

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Online publication date: 06 August 2010

To cite this Article Kedney, P. J. and Leslie, F. M.(1998) 'Switching in a simple bistable nematic cell', Liquid Crystals, 24: 4, 613 — 618
To link to this Article: DOI: 10.1080/026782998207091
URL: http://dx.doi.org/10.1080/026782998207091

LIQUID CRYSTALS, 1998, VOL. 24, NO. 4, 613–618

Switching in a simple bistable nematic cell

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(*Received 17 October 1997; accepted 8 November 1997*)

This paper presents a theoretical examination of a nematic cell bounded by bistable weakly anchored surfaces. Switching between the two resultant equilibrium states is achieved by means of an electric field. In setting up the problem we identify a parameter whose measurement is important to the development of any device which relies upon weak anchoring. The results of this work are discussed in relation to the measurement of this constant.

1. Introduction

Experimental work on surface effects in nematic liquid crystals has shown that at the interface between suitably treated glass plates and a nematic the director may, in the absence of external effects, take one of two easy directions which we shall label $\mathbf{n}_0$ and $\mathbf{n}_1$ (see for example ref. [1]). In this article we consider a nematic within a Cartesian coordinate system such that the liquid crystal is bounded by two such identical bistable surfaces at $z=0$ and $z=h$. For present purposes it is convenient to assume that the easy directions lie within the plane of the plates and so one may choose the coordinate system such that

$$\mathbf{n}_0 = \mathbf{e}_1. \tag{1}$$

The second easy direction, $\mathbf{n}_1$, is then described by a constant angle ϕ_0:

$$\mathbf{n}_1 = (\cos\phi_0, \sin\phi_0, 0). \tag{2}$$

Since the directions $\mathbf{n}$ and $-\mathbf{n}$ are equivalent for nematics it is only necessary to consider $|\phi_0| < \pi/2$. By suitably aligning the plates with respect to each other it is possible to create a simple bistable cell, as depicted in figure 1. This type of cell has two uniform equilibrium states: state A in which the director is everywhere parallel to $\mathbf{n}_0$ and state B where the director is everywhere parallel to $\mathbf{n}_1$.

If we take the initial configuration to be state A, then one can reorientate the director by applying an electric field parallel to the bounding plates:

$$\mathbf{E} = E(\cos\phi_E, \sin\phi_E, 0) \tag{3}$$

where E and ϕ_E are constant. Since the dielectric response of a nematic is insensitive to the sign of $\mathbf{E}$, it is necessary to consider only $E>0$ and $|\phi_E| < \pi/2$ explicitly. Taking

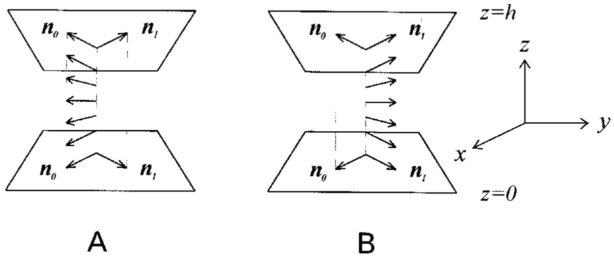

Figure 1. A schematic of the simple bistable cell showing states A and B which represent $\mathbf{n}(z, t) \equiv \mathbf{n}_0$ and $\mathbf{n}(z, t) \equiv \mathbf{n}_1$, respectively.

the dielectric anisotropy to be positive (which is usually true for nematics), one then expects the director to try to align with $\pm\mathbf{E}$. Thus, the initial and expected final director orientations are both parallel to the bounding plates and so it is natural to expect $\mathbf{n}$ to remain in the xy plane throughout the switching process. Hence we set

$$\mathbf{n} = (\cos\phi, \sin\phi, 0) \tag{4}$$

where ϕ is a function of z and time, t, only.

In §2 we derive the relevant equations governing the dynamic behaviour of the cell. The nematic continuum theory reduces to a single bulk equation plus one equation coupling this to the weakly anchored surfaces. To model the weak anchoring dynamics it is necessary to introduce one new parameter on top of those commonly used for bulk effects and surface energies. In §5 we describe how the results presented below may be used to measure this parameter.

The simplest form of switching from state A to state B that one may consider occurs when $\mathbf{E}$ is parallel to $\mathbf{n}_1$. In this case the electric field is forcing the cell into state B. If this electrical distortion is great enough, the cell adopts equilibrium state B after any electric field

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has been removed. This switching process is examined in § 3.

The various cases that arise when considering an electric field applied non-parallel to $\mathbf{n}_1$ are discussed in § 4. Examples are given in two cases which permit switching between equilibrium states, and the effect of the surface energy upon the electrical distortion is noted.

2. Continuum equations

In the Ericksen–Leslie continuum theory for nematics the relevant balance of angular momentum can be written [2, 3]

$$\frac{\partial}{\partial z}\left(\frac{\partial W}{\partial \dfrac{\partial \phi}{\partial z}}\right)-\frac{\partial W}{\partial \phi}-\frac{1}{2}\frac{\partial \Delta}{\partial \dfrac{\partial \phi}{\partial t}}+\frac{\partial \psi}{\partial \phi}=0 \tag{5}$$

where W, the Frank distortion energy, is [4]

$$\begin{aligned} W &= \frac{1}{2}K_1(\nabla\ \mathbf{n})^2+\frac{1}{2}K_2(\mathbf{n}\ \nabla\wedge\mathbf{n})^2+\frac{1}{2}K_3(\mathbf{n}\wedge\nabla\wedge\mathbf{n})^2 \\ &\quad +(K_4+K_2)\nabla\ [(\mathbf{n}\ \nabla)\mathbf{n}-(\nabla\ \mathbf{n})\mathbf{n}] \\ &= \frac{1}{2}K_2\left(\frac{\partial \phi}{\partial z}\right)^2. \end{aligned} \tag{6}$$

Here ψ is the electrical energy

$$\begin{aligned} \psi &= \frac{1}{2}\varepsilon_0[\varepsilon_a(\mathbf{n}\ \mathbf{E})^2+\varepsilon_{\perp}E^2] \\ &= \frac{1}{2}\varepsilon_0[\varepsilon_a E^2\cos^2(\phi_E-\phi)+\varepsilon_{\perp}E^2] \end{aligned} \tag{7}$$

and Δ is the rate of viscous dissipation per unit volume. Given the special form of $\mathbf{n}$ [see equation (4)], it is possible to balance linear momentum in the absence of bulk flow by correctly assigning the arbitrary pressure that arises on account of incompressibility [3, 5]. In the absence of bulk flow Δ becomes

$$\begin{aligned} \Delta &= \gamma_1\left(\frac{\partial \mathbf{n}}{\partial t}\ \frac{\partial \mathbf{n}}{\partial t}\right) \\ &= \gamma_1\left(\frac{\partial \phi}{\partial t}\right)^2. \end{aligned} \tag{8}$$

In these equations the K_i are elastic constants, $\varepsilon_0\varepsilon_a$ is the dielectric anisotropy [6] and γ_1 has the dimensions of viscosity. By inserting equations (6)–(8) into (5) one obtains the following partial differential equation that governs director orientation in the bulk:

$$K_2\frac{\partial^2\phi}{\partial z^2}+\frac{1}{2}\varepsilon_0\varepsilon_a E^2\sin(2\phi_E-2\phi)=\gamma_1\frac{\partial \phi}{\partial t}. \tag{9}$$

Mathematically we model bistable weak anchoring by assigning an energy for the orientation of the director at the surface with respect to that surface. This surface energy per unit area, W_s, must then possess minima at both $\mathbf{n}_0$ and $\mathbf{n}_1$. For W_s to be sufficiently invariant it must be at least a quartic function of the director $\mathbf{n}(0, t)$ or $\mathbf{n}(h, t)$, as required. For a problem of bistable weak anchoring in which the director remains within a given plane (in our case the plane of the plates), the works of Sergan and Durand [7], or Nobili and Durand [8] give

$$W_s = A\sin^2\phi\sin^2(\phi_0-\phi) \tag{10}$$

where A is a surface elastic constant with the dimensions of force/length. We follow Sergan and Durand [7] in writing this constant as

$$A=\frac{K_2}{l_{K_2}} \tag{11}$$

and so discuss the anchoring strength in terms of the extrapolation length l_{K_2}.

To determine interfacial effects we employ a modified form of Jenkins and Barratt's surface equations [9]. In particular, on the bounding plates we set

$$\mp\frac{\partial W}{\partial \dfrac{\partial \phi}{\partial z}}+\frac{\partial W_s}{\partial \phi}+\eta\frac{\partial \phi}{\partial t}=0,$$

$$-\ \text{on}\ z=0\ \text{and}+\text{on}\ z=h, \tag{12}$$

where η is what we shall refer to as the surface viscosity. Since this quantity has the dimensions of viscosity × length it is natural to write

$$\eta=\gamma_1 l_{\gamma_1} \tag{13}$$

where the length, l_{γ_1}, is a measure of the strength of the drag experienced by the director at the boundaries as compared with that in the bulk. Substitution of equation (10) into (12) then determines the boundary conditions,

$$\begin{aligned} &\mp K_2\frac{\partial \phi}{\partial z}+2\frac{K_2}{l_{K_2}}\sin\phi\sin(\phi_0-\phi)\sin(\phi_0-2\phi) \\ &\quad +\gamma_1 l_{\gamma_1}\frac{\partial \phi}{\partial t}=0, \qquad \text{on}\ z=0,\ h. \end{aligned} \tag{14}$$

Because the z derivative of ϕ changes sign in opposing halves of the cell, this boundary condition is in fact symmetric.

Unfortunately, due to the non-linearity of the governing equations it is not possible to construct an analytic solution. Further, in order to model the possibility of switching from state A to state B we must allow ϕ to vary by at least ϕ_0, and since ϕ_0 is not necessarily small we cannot linearize our equations to obtain even an approximate analytic solution. For these reasons it is necessary to construct a numerical solution to equation (9). This may be done with the boundary conditions (14) and the initial condition

$$\phi(z, 0) \equiv 0 \tag{15}$$

which selects state A. Before proceeding with a numerical investigation, one must assign values to the parameters which have been introduced. Unless otherwise stated, all parameters are in cgs units and all angles are in radians. For convenience some parameters are given as μm, μs or ms where $1\,\mu\text{m} = 10^{-4}\,\text{cm}$, $1\,\mu\text{s} = 10^{-6}\,\text{s}$ and $1\,\text{ms} = 10^{-3}\,\text{s}$. For the bulk parameters a reasonable choice is

$$K_2 = 3 \times 10^{-7}, \qquad \varepsilon_0 \varepsilon_a = 1{\cdot}2 \times 10^{-5}, \qquad \gamma_1 = 0{\cdot}5 \tag{16}$$

these being taken from refs [10] and [11] to represent nematic 5CB at 23°C. For the surface energy we set [7]

$$l_{K_2} = 7{\cdot}2 \times 10^{-5}. \tag{17}$$

Experimentally, values for h and ϕ_0 may be measured and E is easily controlled. Our choice for h and ϕ_0 is

$$h = 1\,\mu\text{m}, \qquad \phi_0 = 0{\cdot}8\ \text{rad}. \tag{18}$$

Therefore there is only one unknown parameter in the system, l_{γ_1}, which is a measure of the speed at which the director reorientates at a weakly anchored surface. As such, the measurement of l_{γ_1} is important to the development of bistable weak anchoring devices such as those proposed by Barberi *et al.* [12, 13]. The possibility of using a simple bistable cell geometry to measure l_{γ_1} is examined in more detail in § 5. To proceed numerically with the problem at hand a tentative choice for this parameter is

$$l_{\gamma_1} = 10^{-5}. \tag{19}$$

3. Application of an electric field parallel to $\mathbf{n}_1$

In this section we examine the response of a cell initially in state A to an electric field applied parallel to $\mathbf{n}_1 (\phi_E = \phi_0)$. When this is the case, the director tries to align itself with $\mathbf{n}_1$ in the bulk. If the electric field is strong enough for the bulk distortion to drag the surface orientation past the maximum of W_s at $\phi_0/2$ then the surface orientation also wishes to align with $\mathbf{n}_1$. Thus, the application of a sufficiently strong electric field causes the sample to switch from state A to state B:

$$\lim_{t \to \infty} \phi(z, t) = \phi_0. \tag{20}$$

This switching is demonstrated in figure 2 which shows a numerical solution of the problem when we choose the field strength in our example to be such that

$$\varepsilon_0 \varepsilon_a E^2 = 10^4. \tag{21}$$

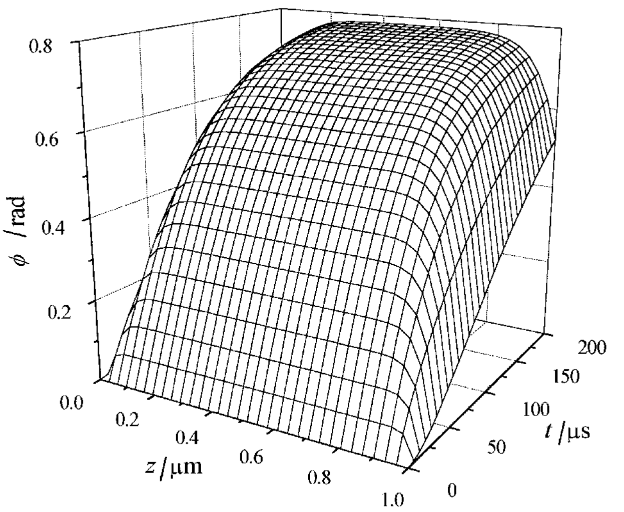

Figure 2. The numerical solution for $\phi(z, t)$ when the nematic, initially in state A, is subjected to an electric field which is both strong enough to break the surface anchoring and parallel to $\mathbf{n}_1$.

The solution shown in figure 2 is typical in that it is symmetric about the centre of the sample and monotonic increasing in z between $z = 0$ and $z = h/2$. The weak anchoring dynamics are evident at both boundaries and it is seen that the director reorientation lags behind that of the bulk. The quantitative measure of this drag is of course l_{γ_1}: the larger the value of l_{γ_1}, the greater the drag at the plates.

It is clear from figure 2 that ϕ simply increases from 0 to 0·8 as time increases. For this reason an experimental observation of the depicted switching process reveals little quantitative information about l_{γ_1}. More information may be gained by subsequently removing the electric field and observing the results, this is discussed in more detail in § 5. Figure 3 shows the relaxation from the final state depicted in figure 2, that is the parameters are as in equation (16) and the field is removed after time $t_{\text{off}} = 0{\cdot}2$ ms.

Figure 3 demonstrates that the initial response of the cell is in reduction of the bulk distortion: director reorientation at both the surfaces and in the bulk produces a state in which ϕ is approximately uniform across the sample. When this happens the elastic K_2 term in the boundary conditions (12) becomes much smaller than the surface energy term which will then dictate the behaviour of the cell at the boundaries. The subsequent boundary-driven reorientation provides the means by which the cell may minimize its total energy by relaxing to one of the two equilibrium states A or B. Characteristically, this relaxation process is slower than the field-driven switching.

During the final relaxation, the bulk orientation lags behind the reorientation at the surface which is now

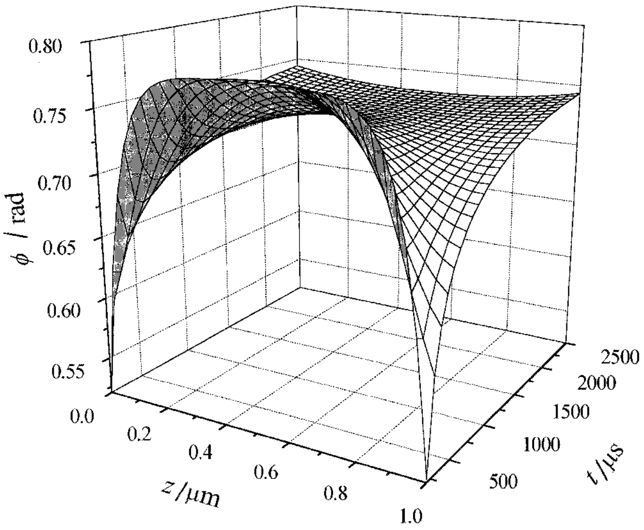

Figure 3. The initial response of the solution $\phi(z, t)$ upon removal of the electric field.

driving the motion. In figure 4 we have shown the subsequent behaviour at the boundaries and centre when the electric field is removed and the sample then relaxes towards equilibrium. State B is attained after approximately 20 ms.

4. Application of fields non-parallel to n_1

When considering a more general problem in which $\phi_E \neq \phi_0$ there are several cases to consider. For example, if $0 < \phi_E < \phi_0/2$, the bulk distortion is never able to force the surface orientation past $\phi_0/2$. Hence, for this case the sample always relaxes to state A when the field is removed. Therefore no switching is possible.

In each case that does permit switching it is a necessary condition for a transition that the applied field is at least strong enough to break the surface anchoring. This is sufficient only if the electric field is left on long enough.

When $\phi_0/2 < \phi_E < \phi_0$ and a sufficiently strong electric field is left on long enough, the director tends to a state where it is approximately equal to ϕ_E in the bulk and slightly greater than ϕ_E at the boundaries. The distortion at the boundaries is caused by the fact that, since the electric field has pulled the surface orientation past $\phi_0/2$, the surface energy decreases if the surface orientation increases towards ϕ_0. Any such decrease is of course played off against the rise in bulk plus electric energy due to the distortion near the surfaces. This type of behaviour is depicted in figure 5 in which we have set $\phi_0 = 1{\cdot}5$, $\phi_E = 1$ and $t_{\text{off}} = 2{\cdot}5$ ms.

Switching is also possible for $0 < \phi_0 < \phi_E < \pi/2$. In this case the electrical distortion tries to pull the director past ϕ_0. Clearly this allows the surface orientation to exceed $\phi_0/2$ and so permit relaxation to state B. If the field is left on long enough the director field again tends to a distorted state when $\phi(h/2, t) \approx \phi_E$ but at the boundaries ϕ is slightly less than ϕ_E in an attempt to lower the surface energy. This is depicted in figure 6 which was calculated using $\phi_0 = 1$, $\phi_E = 1{\cdot}5$ and $t_{\text{off}} = 2{\cdot}5$ ms.

From the symmetry of the director and its dielectric response, the relaxation of a cell to an electric field applied at angle ϕ_E is the same as when the field is applied at $\phi_E + \pi$. Thus the only other possible cases we need to consider arise when $0 < -\phi_E < \pi/2$. This allows the possibility of switching to state B via a clockwise (in the xy plane) twist. Such a motion occurs if the cell orientation relaxes to $\phi \equiv -(\pi - \phi_0)/2$ and the director tends towards $-\mathbf{n}_1$. A necessary criterion for this to occur is that $0 < (\pi - \phi_0)/2 < -\phi_E < \pi/2$. Note that this type of switching permits a total director twist which exceeds $\pi/2$. If $0 < -\phi_E < (\pi - \phi_0)/2$, no switching is possible.

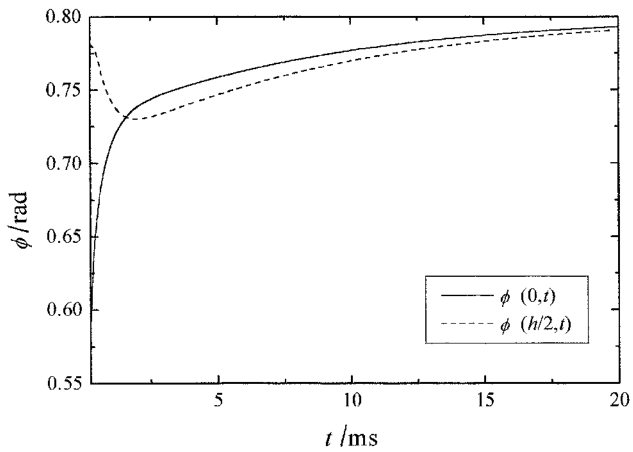

Figure 4. Boundary and centre values of ϕ as sample relaxes towards state B.

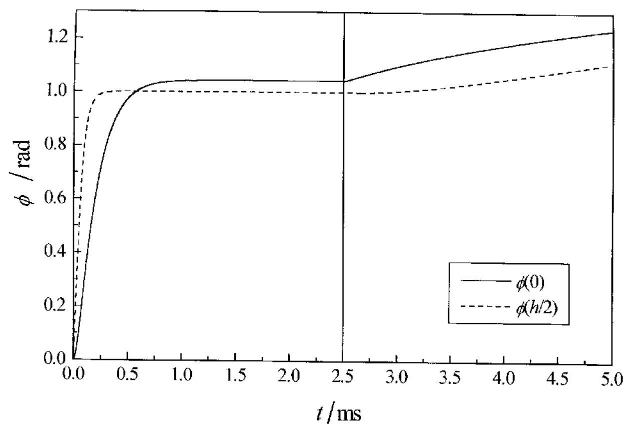

Figure 5. Typical plot of boundary and centre values of $\phi(z, t)$ when $\phi_0/2 < \phi_E < \phi_0$. The plot shows the response before and after the electric field is removed at time $t_{\text{off}} = 2{\cdot}5$ ms. In this example $\phi_E = 1$ and $\phi_0 = 1{\cdot}5$.

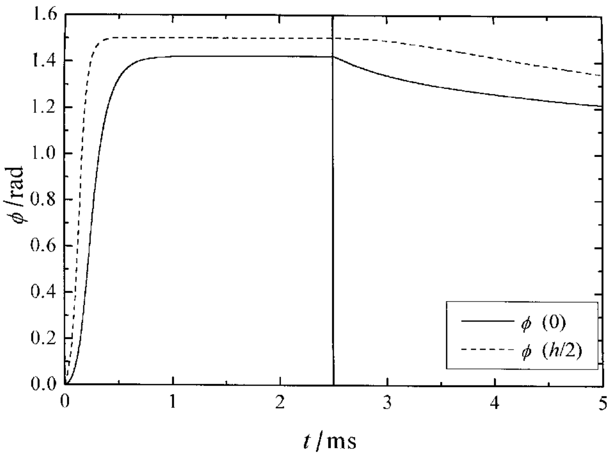

Figure 6. Typical plot of boundary and centre values of $\phi(z, t)$ when $0 < \phi_0 < \phi_E < \pi/2$. In this example $t_{\rm off} = 2{\cdot}5$ ms, $\phi_E = 1{\cdot}5$ and $\phi_0 = 1$.

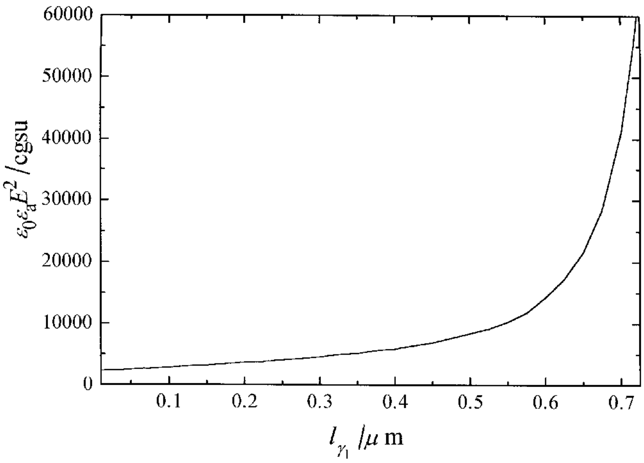

Figure 7. The numerically calculated minimum field strength which permits switching plotted against l_{γ_1}. The parameters used here are as in equations (16)–(18) and $t_{\rm off} = 200\,\mu$s.

5. Measurement of l_{γ_1}

The results in this section are discussed with reference to an electric field applied parallel to $\mathbf{n}_1$. Similar results may be obtained from any of the cases discussed in §4 which permit switching from state A to state B.

From the results of §3 it is clear that it is the degree to which the electric field distorts the sample before it is removed that will determine whether or not the sample switches between equilibrium states. The electric field produces a greater distortion if either its strength is increased or it is left on for a longer period of time. Thus, when considering an electric field that is left on for, say, 200 µs there must be a minimum field strength above which the cell relaxes to state B. Given that l_{γ_1} is the only unknown in the problem, an experimental observation of this minimum field strength should then, in conjunction with appropriate numerical work, allow the measurement of this constant.

In figure 7 we have plotted the numerically calculated minimum field strength for switching to state B against l_{γ_1}. It can be seen that there is a minimum field strength below which switching does not occur irrespective of the value of l_{γ_1}. This represents the fact that the applied field must be strong enough to break the surface anchoring. Equivalently, l_{γ_1} could be measured by applying an electric field of a specified strength and measuring the minimum amount of time for which it must be applied in order to induce switching. This approach would be marginally less favourable as one would need to know in advance the minimum field strength that permits switching.

A simple dimensional analysis proves useful for the measurement of l_{γ_1}. The parameters and associated dimensions in our problem are

$$[h] = L, \quad [l_{\gamma_1}] = L, \quad [l_{K_2}] = L, \quad [t_{\rm off}] = T,$$
$$[K_2] = \frac{ML}{T^2}, \quad [\gamma_1] = \frac{M}{LT}, \quad [\varepsilon_0 \varepsilon_a E^2] = \frac{M}{LT^2}. \tag{22}$$

Since there are seven parameters and three dimensions present, we are examining a relationship between four dimensionless variables,

$$\Pi_1 = f(\Pi_2, \Pi_3, \Pi_4). \tag{23}$$

We choose

$$\Pi_1 = \frac{\varepsilon_0 \varepsilon_a E^2 t_{\rm off}}{\gamma_1} \tag{24}$$

$$\Pi_2 = \frac{K_2}{\varepsilon_0 \varepsilon_a E^2 l_{K_2}^2} \tag{25}$$

$$\Pi_3 = \frac{h}{l_{\gamma_1}} \tag{26}$$

$$\Pi_4 = \frac{h}{l_{K_2}}. \tag{27}$$

For typical values these give

$$\frac{K_2}{\varepsilon_0 \varepsilon_a E^2 l_{K_2}^2} = O(10^{-4}), \tag{28}$$

$$\frac{h}{l_{\gamma_1}} = O(10), \tag{29}$$

$$\frac{h}{l_{K_2}} = O(1). \tag{30}$$

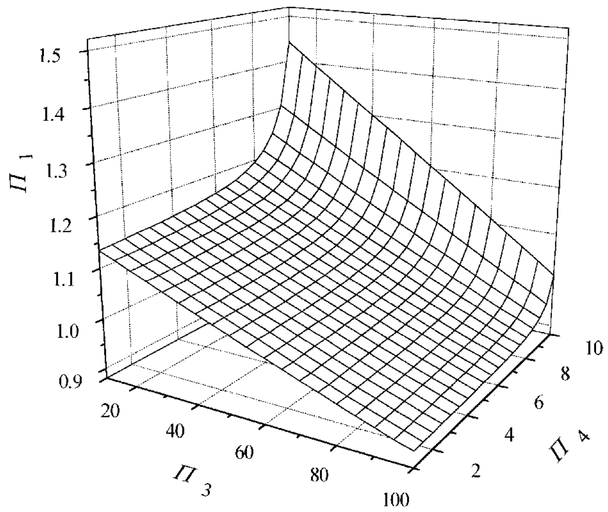

Figure 8. Critical values of Π_1 above which switching from state A to B will occur.

Clearly we wish to examine Π_1 versus Π_3 and Π_4. From the above it seems that Π_2 has a lesser effect on the switching process. As a result of this we may produce figure 8. Since we know the values of the material parameters this plot may be used to determine a precise relationship between critical values of E, t_{off} and l_{γ_1} over a wide range of values.

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